10E-36. The volume of the Grand Canyon is 2500 cubic miles. If this volume were spread uniformly over the entire surface of the earth, how thick would the added layer be? $\qquad$
$\qquad$ in

$$
\begin{aligned}
& \mathrm{SA}_{\text {earth }}=4 \pi(3960)^{2}=1.97 \times 10^{8} \mathrm{mi}^{2} \\
& \left(2500 \mathrm{mi}^{3} / 1.97 \times 10^{8} \mathrm{mi}^{2}\right) \times(5280 \mathrm{ft} / \mathrm{mi}) \times(12 \mathrm{in} / \mathrm{ft})=. \mathbf{8 0 4}
\end{aligned}
$$

11A-36. Electric rail guns accelerate objects at incredible rates. A
$50-\mathrm{g}$ armature is accelerated horizontally from rest to $1.5 \mathrm{~km} / \mathrm{s}$ over a
2 -meter distance. How much energy is necessary to accomplish this?
Energy is the product of the applied force and the distance traveled. --- 36= $\qquad$ kJ

$$
\begin{aligned}
& \mathrm{E}=\mathrm{F} \times \mathrm{d}\{\text { work }\} \quad \mathrm{v}^{2}=\mathrm{v}_{0}^{2}+2 \mathrm{a}\left(\mathrm{x}-\mathrm{x}_{0}\right) \\
& \mathrm{a}=\frac{\mathrm{v}^{2}-\mathrm{v}_{0}^{2}}{2\left(\mathrm{x}-\mathrm{x}_{0}\right)}=\frac{1500^{2}-0}{2(2)}=562,500 \mathrm{~m} / \mathrm{s}^{2} \\
& \mathrm{~F}=\mathrm{ma} \\
& \mathrm{E}=\operatorname{mad} \\
& \mathrm{E}=(.050)(562,500)(2)=56,250 \mathrm{~J}=\mathbf{5 6 . 3} \mathbf{~ k J}
\end{aligned}
$$

11B-36. A person can jump 4 ft vertically on earth. For the same effort, defined as identical initial velocity, how far could they jump on the moon, if the gravitational acceleration is $16.7 \%$ that of earth?--- $36=$ $\qquad$ ft

Height inversely proportional to gravity: $4 \times 1=\mathrm{h} \times .167 \quad \mathrm{~h}=4 / .167=\mathbf{2 4 . 0}$
11D-37. On a clear day at the 102 meter tall observation deck of the Atomium in Brussels, Belgium, one can just make out the cathedral of Antwerp on the horizon. How far is it from Brussels to Antwerp? ----- 37= $\qquad$ mi

$$
\begin{aligned}
& \mathrm{r}=3960 \mathrm{mi} \quad \mathrm{~d}=102 \mathrm{~m}=.06337 \ldots \mathrm{mi} \\
& \cos \theta=\mathrm{r} /(\mathrm{r}+\mathrm{d})=.32416 \ldots \circ \Rightarrow .0045677 \ldots \mathrm{rad} \\
& \mathrm{~s}=\theta \mathrm{r}=\mathbf{2 2 . 4}
\end{aligned}
$$



11F-38. A trapeze artist swings from one $35-\mathrm{ft}$ long trapeze (or swing) to another trapeze. She releases one trapeze at a $30^{\circ}$ angle relative to the vertical. She flies through the air, catching the other trapeze 18 ft away at the same elevation. What was her release velocity? --------- $38=$ $\qquad$ fps

$$
\begin{aligned}
& d_{h_{\max }}=\frac{v^{2} \sin 2 \theta}{g} \\
& 18=\frac{v^{2} \sin 60^{\circ}}{(32.174)} \quad \text { v }=\mathbf{2 5 . 9}
\end{aligned}
$$



## Page 2

11I-36. A carton crushes if its impact velocity exceeds 80 mph . If the carton is thrown vertically upward from a $60-\mathrm{ft}$ tall building, and it just crushes when it hits the ground, what was the initial velocity (up is positive)? 36= $\qquad$ mph

$$
\begin{aligned}
& v^{2}=v_{0}^{2}+2 a\left(y-y_{0}\right) \\
& {[(80)(88 / 60)]^{2}=v_{0}^{2}+2(-32.174)(0-60)}
\end{aligned}
$$

$$
v_{0}=99.53005 \times 60 / 88=67.9
$$

$12 \mathrm{~A}-36$. The mass of the earth is $5.9742 \times 10^{24} \mathrm{~kg}$. What is the percent error in calculating the mass based on an average density of $5.613 \mathrm{~g} / \mathrm{cm}^{3}$ ? --------- $36=$ $\qquad$ \%(SD)
$\mathrm{r}=3960 \mathrm{mi} \times(5280 \mathrm{ft} / \mathrm{mi}) \times(12 \mathrm{in} / \mathrm{ft}) \times(2.54 \mathrm{~cm} / \mathrm{in}) \times(1 \mathrm{~m} / 100 \mathrm{~cm})=6373002.24 \mathrm{~m}$
$5.613 \mathrm{~g} / \mathrm{cm}^{3}=5,613 \mathrm{~kg} / \mathrm{m}^{3} \times(4 / 3) \pi \mathrm{r}^{3}=6.08577462 \ldots \times 10^{24} \mathrm{~kg}=$ calculated mass
(exact, approx.., \%chg): [ $\left.\underline{5.9742} \times 10^{24} \mathrm{~kg}, 6.08577 \ldots \times 10^{24} \mathrm{~kg}, \% \mathrm{chg}\right]=1.8676=1.9$ (2SD)
\{For SD calculation\}:
[(6.08577462 \{4SD\} / $5.9742\{5 S D\})-1] \times 100 \%$
When you divide above, you get $\underline{1.018676077 \text { to 4SD. }}$
When you subtract 1 , you get .018676077 to the thousandths place $\rightarrow .019$
Times 100\% = 1.9 (2SD)
12C-36. What is the closest approach of the line $y=5.5 x+15$ to the origin? $\qquad$ $36=$ $\qquad$

$$
\begin{aligned}
& 5.5 \mathrm{x}-\mathrm{y}+15=0 \quad \rightarrow \quad \mathrm{a}=5.5, \quad \mathrm{~b}=-1, \quad \mathrm{c}=15, \quad \mathrm{x}=0, \quad \mathrm{y}=0 \\
& d=\frac{|a x+b y+c|}{\sqrt{a^{2}+b^{2}}}=\frac{|15|}{\sqrt{(5.5)^{2}+(-1)^{2}}}=2.68
\end{aligned}
$$

12C-38. A motorcycle dare devil rides his motorcycle up a $20^{\circ}$ ramp at 55 mph . The ramp was built using 10 sheets of 8 -ft long plywood. What is the horizontal distance from the end of the ramp to the spot on the ground where the dare devil lands? $38=$ $\qquad$ ft

$$
\begin{aligned}
& 55 \mathrm{mph}(88 / 60)=80.666 \ldots \mathrm{ft} / \mathrm{sec} \\
& \mathrm{y}=\mathrm{y}_{0}+\mathrm{v}_{0} \mathrm{t}+1 / 2 \mathrm{at}^{2} \\
& 0=80 \sin 20^{\circ}+\left(80.66 \sin 20^{\circ}\right) \mathrm{t}+1 / 2(-32.174) \mathrm{t}^{2} \\
& \mathrm{t}=2.4185 \ldots \mathrm{sec} \\
& x=x_{0}+v_{0} t+1 / 2 \mathrm{at}^{2} \\
& x=0+\left(80.6 \cos 20^{\circ}\right)(2.4185 \ldots)+0 \quad x=183
\end{aligned}
$$



## Page 3

12G-37. The US leads the world in generation of municipal waste, generating 760 kg annually per person. How many Olympic-sized swimming pools would be filled by garbage in the US annually? The US population is $307,006,550$, the capacity of a pool is $\underline{2520}$ cubic meters and garbage density is $\underline{0.85} \mathrm{~g} / \mathrm{cm}^{3} .37=$

$$
\begin{equation*}
\frac{(307,006,550)(760 \mathrm{~kg})}{\left(2520 \mathrm{~m}^{3}\right)\left(.85 \mathrm{~g} / 1 \mathrm{~cm}^{3}\right)(1 \mathrm{~kg} / 1000 \mathrm{~g})(100 \mathrm{~cm} / 1 \mathrm{~m})^{3}}=108,928.56=110,000 \quad\{2 \mathrm{SD}\} \tag{SD}
\end{equation*}
$$

12l-37. The world's gross domestic product (GDP) in 2010 was $\$ 62.909274$ trillion. The US GDP was $\$ 14.6578$ trillion. What is the percent error in estimating the world GDP to be four times the US GDP? $37=$ $\qquad$ \%(SD)

$$
\left[\frac{4(14.6578)}{62.909274\{6 S D\}}-1\right] 100 \%=-6.8004\{5 \mathrm{SD}\} \quad .93199 \underline{6} 131-1=-.068004
$$

13A-36. A 55 -gal saltwater fish tank is prepared by adding $1 / 2$ cup of salt to each gallon of fresh water. Over time, 3 gallons of pure water (no salt) evaporated from the tank. To replenish the tank, an additional 8 gallons of saltwater was removed. How many cups of salt should be mixed with fresh water to make up the 11 gallons needed to restore the salt concentration of the tank to the proper level? $\qquad$ cups

$$
1 / 2 \text { cup = } 1 / 32 \mathrm{gal}
$$

Let $\mathrm{x}=\%$ of salt water

$$
(1 / 32)(55)-(0)(3)=x(55-3) \quad \rightarrow \quad x=.033053 \ldots
$$

$$
52-8=44
$$

Let $\mathrm{w}=$ gallons of salt
$x(44)+w(11)=(1 / 32)(55) \quad \rightarrow \quad w=.024038 \ldots$
$\mathrm{w}(11)(16 \mathrm{c} / \mathrm{gal})=4.23$

13A-37. Brad left San Saba on State Highway 190 driving to Iraan, 204 mi away, at 53 mph . Brandon left Iraan 30 min after Brad left, driving to San Saba on the same highway. If they met in Eldorado which is 79 mi from Iraan, what was Brandon's velocity? $\qquad$ mph

$$
53 \mathrm{t}=204-79 \rightarrow \mathrm{t}=2.3585
$$

$$
V(\mathrm{t}-.5)=79
$$

$$
V=42.5
$$

## Page 4

13C-38. A spring elongates 1 in for every 5 lbs of load. Four gallons of coconut oil (density equals $0.92 \mathrm{~g} / \mathrm{cm}^{3}$ ) are hung on the spring which is attached to a frame. However, the container has a leak, losing 10 tablespoons of coconut oil every minute. How long will it take for the container to rise 1.875 in? $\qquad$ $38=$ $\qquad$ hr

```
. }92\textrm{g x (1 kg/ 1000g) }->.002028253 pound
cm}\mp@subsup{}{}{3}\times(1\textrm{ml}/1\mp@subsup{\textrm{cm}}{}{3})\times(1\textrm{L}/1000\textrm{mL})->.0002641 gal 7.6777\ldots lb/gal
F=(constant)(distance) F=kx 5=k(1) k=5
F=(5)(1.875 in) = 9.375 lb
9.375 lb / 7.6777...lb/gal = 1.221057...gal
```

$(1.221057 \ldots \mathrm{gal})(128 \mathrm{oz} / \mathrm{gal})(2 \mathrm{Tbsp} / \mathrm{oz})(1 \mathrm{~min} / 10 \mathrm{Tbsp})(1 \mathrm{hr} / 60 \mathrm{~min})=.521$

13D-37. A cube of iron weighs 40 lbs and rests on a table. The cube is pushed which causes it to rotate with one edge always in contact with the table, eventually flipping it onto an adjacent face. How much energy must be applied to accomplish this? Energy is the product of the cube weight and the change in the vertical distance of its centroid which is the center of the cube. The density of iron is $7.86 \mathrm{~g} / \mathrm{cm}^{3}$. $\qquad$ $37=$ $\qquad$ ft -lbs

$$
\begin{aligned}
& 40 \mathrm{lb} \rightarrow 18.14369 \ldots \mathrm{~kg} \times(1000 \mathrm{~g} / \mathrm{kg})=18,143.69 \ldots \mathrm{~g}\{\mathrm{~A}\} \\
& \{\mathrm{A}\} / 7.86=2308.358 \ldots \mathrm{~cm}^{3}\{\mathrm{~B}\} \\
& \sqrt[3]{\mathrm{B}} \mathrm{~cm} \times(1 \mathrm{in} / 2.54 \mathrm{~cm}) \times(1 \mathrm{ft} / 12 \mathrm{in})=.433596 \ldots\{\mathrm{C}\} \\
& \mathrm{h}=\frac{\mathrm{C} \sqrt{2}}{2}-\mathrm{C} / 2=.0898\{\mathrm{D}\} \quad\{\mathrm{E}\}=\mathrm{D}(40)=3.59
\end{aligned}
$$

$13 \mathrm{E}-36$. The total length of active track in the New York subway system is 842 mi . Howard starts inspecting track at $2000 \mathrm{ft} / \mathrm{hr}$. After 800 hr inspecting, Howard still works, but Jana starts inspecting different sections at $2500 \mathrm{ft} / \mathrm{hr}$.
How many hours will Jana work if they completely finish inspecting all the track?- 36= $\qquad$ hr

```
r}\mp@subsup{\textrm{r}}{1}{}\mp@subsup{t}{1}{}+\mp@subsup{r}{2}{}\mp@subsup{t}{2}{}=
2000(t+800)+2500t = 842(5280)
t = 632
```

13F-38. A target is dropped from a 1900 ft tall tower. A bullet is fired from the ground straight up towards the falling target but with a time delay of $t$ seconds. If the bullet initial velocity was 1800 mph , what is t if the bullet hits the target at an elevation of 600 ft ? $\qquad$ $38=$ $\qquad$ s

$$
\begin{aligned}
& y=y_{0}+v_{0} t+1 / 2 a t^{2} \\
& 600=1900+0+1 / 2(-32.174) t^{2} \quad t=8.98947 \ldots[A] \\
& 1800(88 / 60)=2640 \mathrm{ft} / \mathrm{s} \\
& A-B=8.76
\end{aligned}
$$

## Page 5

13G-36. A company can buy a bracket for $\$ 5.70$. They alternatively consider buying a bracket-making machine. The machine costs $\$ 12,000$, labor and electricity to operate the machine is $\$ 30 / \mathrm{hr}$, and the material cost for one bracket is $\$ 0.75$. The machine makes 200 brackets $/ \mathrm{hr}$. What is the minimum number of brackets produced for which it will be cheaper for the company to buy the machine and make their own brackets rather than purchasing brackets? ------ 36= $\qquad$ integer

$$
\begin{aligned}
& 12,000+30(\mathrm{n} / 200)+.75 \mathrm{n}=5.70 \mathrm{n} \\
& \mathrm{n}=2500 \quad \text { (breaks even) } \rightarrow 2501
\end{aligned}
$$

14A-38. Two persons each hold the end of a 20 ft long jumping rope. How far apart should they stand if they hold the rope 4.5 ft off the ground, and the middle of the rope just touches the ground? Assume the arc formed by the jumping rope is circular. -------------------------------------------------------------------------38= $\qquad$ ft

$$
\begin{aligned}
& s=\theta r \quad 10=\theta r \quad \cos \theta=x / r \\
& r=x+4.5 \quad 10=\theta(x+4.5) \\
& \theta=10 /(x+4.5) \quad \cos \theta=x /(x+4.5) \\
& \cos (10 /(x+4.5))=x /(x+4.5) \\
& \text { nsolve } \mid x>0 \quad x=5.7586 \ldots \quad \theta=.97478 \ldots \\
& \tan \theta=w / x \quad w=8.48989 \ldots \quad 2 w=17.0
\end{aligned}
$$



14B-37. A projectile is fired from Odessa to Midland, 20.1 mi away, at a release angle of $49^{\circ}$. What is the projectile maximum elevation during flight? -----37= $\qquad$ mi
$(20.1)(5280)=\left(v^{2} \sin \left[2\left(49^{\circ}\right)\right]\right) / 32.174$
$v=1,856.91117 . .$.
$d_{v \max }=\left(v^{2} \sin ^{2} 49^{\circ}\right) / 2(32.174) \times(1 \mathrm{mi} / 5280 \mathrm{ft})=5.78$

## Alternate solution:

$$
\tan \theta=\left[4\left(\mathrm{~d}_{\mathrm{v} \text { max }}\right)\right] / \mathrm{d}_{\mathrm{h} \max } \quad \tan \left(49^{\circ}\right)=\left[4\left(\mathrm{~d}_{\mathrm{v} \max }\right)\right] / 20.1 \quad\left(\mathrm{~d}_{\mathrm{v} \max }\right)=5.78
$$

14C-38. Atoms travel a distance $x$ through a solid object according to the Arrhenius equation, $x \approx \sqrt{D_{0} t \exp \left(\frac{-Q}{R T}\right)}$, where $D_{0}$ is a constant, $t$ is the elapsed time, Q is the activation energy, R is the universal gas constant [1.987 $\mathrm{cal} /(\mathrm{mol} \cdot \mathrm{K})]$, and T is absolute temperature. Calculate Q if an atom diffuses a distance of 1 mm in 10 s at $800^{\circ} \mathrm{C}$ or in 500 s at $700^{\circ} \mathrm{C}$.--------------------------------38=--3-- $\qquad$ $\mathrm{cal} / \mathrm{mol}$

$$
\begin{aligned}
& .001=\sqrt{\mathrm{D}_{(10)} \mathrm{e}^{\left(\frac{-\mathrm{Q}}{(1.987(800+273.13)}\right)}} \quad .001=\sqrt{\mathrm{D}_{(500)} \mathrm{e}^{\left(\frac{-\mathrm{Q}}{1.987(700+273.13)}\right)}} \\
& \text { Solve: } \mathrm{e}^{\left(\frac{-\mathrm{x}}{1.987(1073.13)}\right)}=50 \mathrm{e}^{\left(\frac{-\mathrm{x}}{1.987(973.13)}\right)} \quad \mathrm{x}=\mathbf{8 1 , 2 0 0}
\end{aligned}
$$

## Page 6

14D-37.What is the probability of a monkey typing Shakespeare's play,
MacBeth? The play has 99,110 characters, and there are 48 type-able keys on a keyboard. Assume the monkey presses keys randomly with equal probability, and it has the patience to type the entire play $37=$ $\qquad$

$$
\begin{aligned}
& (1 / 48)^{99110} \quad 99110 \log (1 / 48)=-166,627.819 . . \\
& \text { Add: } 166,628+(-166,627.819 \ldots)=.180963 \\
& 10^{.180963}=1.52 \rightarrow 1.52 \times 10^{-166628}
\end{aligned}
$$

14D-38. A golfer uses a 6 iron to tee off. The golf ball initial velocity is
70 mph , and the 6 iron has a loft angle of $29^{\circ}$ (relative to the horizontal). How
far does the ball travel before hitting the ground?
$38=$ $\qquad$ yd

$$
\mathrm{d}_{\mathrm{h}_{\max }}=\frac{\left(70 \times \frac{88}{60}\right)^{2} \sin \left(58^{\circ}\right)}{32.174} \mathrm{ft} \times \frac{1 \mathrm{yd}}{3 \mathrm{ft}}=92.6
$$

14F-37. A "square" of roofing shingles cover 100 sq. ft of roof. How many squares are needed to roof a simple, A-frame house that is 40 ft wide by 65 ft long? Assume the roof is sloped at $18.4^{\circ}$ relative to the horizontal, the eaves overhang the house by 2 ft all the way round its periphery, and that $5 \%$ extra must be purchased to account for trimming loss. $\qquad$ $37=$ $\qquad$ integer

$$
\begin{aligned}
& \cos 18.4^{\circ}=20 / d \quad d=21.0775 \ldots \\
& d+2=23.0775 \ldots \\
& A=2(23.0775 \ldots)(65+4)=3184.704 \ldots \\
& (1.05) A / 100=33.4 \rightarrow 34
\end{aligned}
$$



14F-38. What is positive $b$ if the right triangle formed by the $x$ - and $y$-axes and the line $y=4 x+b$ has an area of $1200 ?$ $\qquad$ $38=$


$$
\begin{array}{lll}
A=1 / 2 w b & -1200=1 / 2 w b \\
b=y-4 x & b=0-4 w & w=b /(-4)
\end{array}
$$

$$
-1200=1 / 2(b /-4) b \quad b=98.0
$$

14G-37. What is the weight of a car if its mass is 75 slugs? A 1 -slug object is accelerated by $1 \mathrm{ft} / \mathrm{s}^{2}$ when 1 lb (force) is applied. $\qquad$ $37=$ $\qquad$ lbs

$$
F_{w}=m g=75(32.174)=2410
$$

## Page 7

14G-38. A top-fuel dragster races on a 0.25 mi straight track. It accelerates from rest to 325 mph in the first 500 ft and then finishes the race at constant velocity. What is the posted time for the race?

$$
\begin{aligned}
& 325(88 / 60)=476.66 \ldots \mathrm{ft} / \mathrm{s} \\
& v^{2}=v_{0}{ }^{2}+2 \mathrm{ax} \\
& (476.66 . .)^{2}=2 \mathrm{a}(500) \quad \rightarrow \quad \mathrm{a}=227.2111 \ldots \mathrm{ft} / \mathrm{s}^{2} \\
& v=v_{0}+\mathrm{at} \rightarrow \quad \mathrm{t}=\left(\mathrm{v}-\mathrm{v}_{0}\right) / \mathrm{a} \\
& (476.66 \ldots-0) / 227.2111 \ldots=2.0979 \ldots \mathrm{sec}(\mathrm{C}) \\
& 5280 / 4=1320 \mathrm{ft} \quad 1320-500=820 \mathrm{ft} \quad 820 \mathrm{ft} / 476.66 \ldots \mathrm{ft} / \mathrm{s}=1.72027 \ldots \mathrm{sec}(\mathrm{D}) \\
& C+D=3.82
\end{aligned}
$$

$14 \mathrm{H}-38$. An artillery shell is fired at an angle of $33^{\circ}$ relative to horizontal but falls 300 ft short of the target. The angle is adjusted to $41^{\circ}$ to hit the target. What is the projectile initial velocity? $\qquad$ mph

$$
\begin{aligned}
& (x-300)=\frac{v^{2} \sin 66^{\circ}}{32.174} \quad x=\frac{v^{2} \sin 82^{\circ}}{32.174} \\
& \frac{v^{2} \sin 82^{\circ}}{32.174}-300=\frac{v^{2} \sin 66^{\circ}}{32.174}
\end{aligned}
$$

Solve: $v=354.692 \ldots \mathrm{ft} / \mathrm{s}(60 / 88)=242$
14I-36. Every Formula 1 racing car can decelerate from 100 mph to zero and then accelerate back to 100 mph , all in less than 5 s . Assuming deceleration and acceleration are equal, what minimum, positive acceleration does this represent? $36=$ $\qquad$ $\mathrm{ft} / \mathrm{s}^{2}$

$$
\begin{aligned}
& v=v_{0}+a t \quad a=\left(v-v_{0}\right) / t \quad 100 \mathrm{mph}(22 / 15)=146.666 \ldots \mathrm{ft} / \mathrm{s} \\
& a=(146.66 \ldots-0) / 2.5 \mathrm{~s}=58.7
\end{aligned}
$$

14I-37. A circular saw has a 6 -in diameter blade. It is used to cut a 2 -in diameter rod into two pieces. When the blade edge reaches the center of the
 $\qquad$ $i n^{2}$

$$
\begin{aligned}
& (x-0)^{2}+(y-3)^{2}=9 \\
& y-3= \pm \sqrt{9-x^{2}} \quad y=3 \pm \sqrt{9-x^{2}} \\
& y_{1}=3-\sqrt{9-x^{2}} \quad x^{2}+y^{2}=1 \quad y_{2}=\sqrt{1-x^{2}}
\end{aligned}
$$



Solve $y_{1}=y_{2}$

$$
x=\frac{\sqrt{35}}{6}
$$

$$
2 \int_{0}^{\sqrt{35}}\left[y_{2}-y_{1}\right] d x=A
$$

$\pi(1)^{2}-A=1.68$

Page 8

```
10B-50.
```

    HEMISPHERE WITH
        HEMISPHERICAL CAVITY
    

Total Surface Area $=19.7$
$\mathrm{SA}=3 \pi \mathrm{r}_{1}{ }^{2}-\pi \mathrm{r}_{2}{ }^{2}+2 \pi \mathrm{r}_{2}{ }^{2}$
$3 \pi R^{2}-\pi(.478)^{2}+2 \pi(.478)^{2}=19.7$
$3 \pi \mathrm{R}^{2}+\pi(.478)^{2}=19.7$
$\mathrm{R}=1.419$
$\mathrm{D}=\mathbf{2 . 8 4}$
$10 B-50=$ $\qquad$

10I-50.

RECTANGULAR SOLID

$10 \mathrm{I}-50=$ $\qquad$



$a=\sqrt{577^{2}+384^{2}}=693.098 \ldots$
$\mathrm{b}=\sqrt{848^{2}+577^{2}}=1025.686 \ldots$
$c=\sqrt{384^{2}+1025.686 .^{2}}=1095.211 \ldots$
$848^{2}=693^{2}+1095^{2}-2(693)(1095) \cos \theta$
$\theta=. \mathbf{8 8 6}$
$\mathrm{V}=1 / 3\left[\mathrm{~A}_{1}+\mathrm{A}_{2}+\sqrt{\mathrm{A}_{1} \mathrm{~A}_{2}}\right] \mathrm{h}$
$A_{1}=1 / 2(6.78)(18.5)=62.715$
$6.78 / 3.39=18.5 / b \quad b=9.25$
$A_{2}=1 / 2(b)(3.39)=15.67875$
$h=26.8$
$V=980$

## Page 9

```
11H-50.
    CONGRUENT CONES, ONE OF WHICH IS
        TRUNCATED
```



```
            Total Volume = ?
11H-50=
```

$\qquad$
$\tan \left(25.2^{\circ}\right)=\mathrm{r}_{1} / 79$ $\mathrm{r}_{1}=37.174 \ldots$

$\mathrm{h}_{\mathrm{f}}=(.6) \mathrm{h}_{\mathrm{c}}=(.6)(79)=47.4$
$\mathrm{r}_{2}=(.4)\left(\mathrm{r}_{1}\right)=14.869 \ldots$
$\mathrm{V}=(1 / 3) \pi \mathrm{r}^{2} \mathrm{~h}_{\mathrm{c}}+(1 / 3) \pi\left(\mathrm{r}_{1}{ }^{2}+\mathrm{r}_{2}{ }^{2}+\mathrm{r}_{1} \mathrm{r}_{2}\right) \mathrm{h}_{\mathrm{f}}$
$V=221000$
or
Find cone, double it, delete missing cone.

## 12B-50.

CUBE AND PYRAMID WITH TRIANGULAR BASE


Cube Volume $=2[$ Pyramid Volume $]$
$12 B-50=$ $\qquad$

Base is equilateral $\Delta$
$B=\frac{(72.9 \sqrt{2})^{2} \sqrt{3}}{4}=4602.414 \ldots$
$V_{\text {cube }}=72.9^{3}=387,420.489$
$\mathrm{V}_{\text {pyramid }}=1 / 2 \mathrm{~V}_{\text {cube }}=193,710.2445=1 / 3(B)(H)$ $H=126.266 \ldots$.
$1 / 2(72.9 \sqrt{2}) h=B \quad h=89.2839 \ldots$
$\operatorname{Sin} \theta=72.9 / \mathrm{h} \quad \theta=54.7356 \ldots$
$\alpha=90-\theta=35.264 \ldots$
$2 / 3 h=59.5226 \ldots \quad\{D\}$
$D \sin (\theta)+H \sin (\alpha)=122$

12C-60.
EQUILATERAL TRIANGLE AND SEMICIRCLE

$\mathrm{A}=$ midpoint
$A B=1.82$
$\tan 30^{\circ}=r /(r+.784)$
$r=1.070964$
$\sin \theta=r / 1.82$
$\theta=36.0465 \ldots$
$\alpha=90-\theta$
= 53.95349...

$\beta=180-(60+\alpha)=66.0465 \ldots{ }^{\circ}$
$\sin \beta / x=\sin 60^{\circ} / 1.82$
$x=1.92 \quad 2 x=3.84$

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13E-50. CONE, CUBE AND CYLINDER

$13 \mathrm{E}-50=$ $\qquad$
$[(1.3)(7.15)]^{2}+[(7.15 / 2)+1.3(7.15)]^{2}=x^{2}$ $x=15.9$

13H-50.
RECTANGULAR SOLID WITH SQUARE FACE AND

$2(68.4)^{2}+4(68.4 x)-2(68.4-2 x)^{2}+4[(68.4-2 x)(x)]$
$=15,000$

$$
x=19.0
$$

Page 11



Hatched Area $=44.5$

Top triangle, $\theta=.45$
square - triangle: $x^{2}-1 / 2 x w=44.5$
$\tan (.45)=w / x$
$x^{2}-1 / 2(x)(x \tan (.45))=44.5$
$x=7.66$
$14 \mathrm{~A}-60=$ $\qquad$ 14B-49.

CYLINDER
$C=$ Center

$A B=46.2 \quad B C=36.4$

$h^{2}+(2 r)^{2}=46.2^{2}$
$h^{2}+r^{2}=36.4^{2}$
solve: $h=32.5$

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14C-49.
$d=\sqrt{50^{2}+25.3^{2}}=56.0365 \ldots$
$s=\frac{50+25.3+d}{2}=65.6682$
$A=\sqrt{s(s-50)(s-25.3)(s-d)}=632.499 \ldots$
$\mathrm{V}_{1}=(1 / 3) \mathrm{A}(21.3)=4,490.7499 \ldots$
$V_{2}=50(25.3)(21.3)=26,944.5$
$\mathrm{V}_{2}-\mathrm{V}_{1}=\mathbf{2 2 , 5 0 0}$
$14 \mathrm{C}-49=$ $\qquad$
14C-60. $\quad$ SEMICIRCLE AND RIGHT TRIANGLES
14D-50.

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$$
\begin{aligned}
& \text { Area }= \sum \frac{x^{2} \sqrt{3}}{4}+\left[\frac{x \sin 32^{\circ}}{\sin 88^{\circ}}\right]^{2} \frac{\sqrt{3}}{4} \\
&+\left[\frac{x \sin 32^{\circ} g \sin 32^{\circ}}{\sin 88^{\circ} g \sin 88^{\circ}}\right] \\
& 277=\frac{\sqrt{3}}{4} \sum_{n=0}^{\infty} x^{2}\left[\frac{\sin 32^{\circ}}{\sin 88^{\circ}}\right]^{2 n}
\end{aligned}
$$

Solve: $\quad 277=\frac{\sqrt{3}}{4} \mathrm{gx}^{2} \mathrm{~g} \sum_{\mathrm{n}=0}^{\infty}\left[\frac{\sin 32^{\circ}}{\sin 88^{\circ}}\right]^{2 n}$
$\mathrm{x}=21.4$

14G-49.


Total Volume $=30$
14G-49 = $\qquad$

$$
\begin{aligned}
& \begin{aligned}
30= & \frac{1}{3} \pi(1.99)\left[\left(\frac{5.16}{2}\right)^{2}+x^{2}+\frac{5.16 x}{2}\right] \\
& \quad+\frac{1}{3} \pi(x)^{2}(3.3)
\end{aligned} \\
& \begin{aligned}
x= & 1.29 \\
2 x= & 2.58
\end{aligned}
\end{aligned}
$$

CIRCLE AND SEGMENT


Hatched Area $=1580$
$\qquad$

$\pi(25.2)^{2}-\left[(25.2)^{2} / 2\right](\theta-\sin \theta)=1580$
$\theta=2.146 \ldots$
$\cos (\theta / 2)=\mathrm{w} / 25.2$
w = 12.0313...
$25.2-\mathrm{w}=13.2$

## Page 14


(2) $\frac{r^{2}}{2}\left(\alpha_{1}-\sin \alpha_{1}\right)+\frac{r^{2}}{2}\left(\alpha_{2}-\sin \alpha_{2}\right)$
$339=(33.45)^{2}(\alpha-\sin \alpha)$
$+(33.45)^{2} / 2[(\pi-2 \alpha)-\sin (\pi-2 \alpha)]$
If $\theta<1$, then $\alpha$ has to be $>1$ and $\alpha=1.205 \ldots$
then $(\pi-\alpha) / 2=.968$

$\sin 38.9^{\circ}=x / 1.96$
$x=1.2308 \ldots$

$\cos 38.9^{\circ}=m / 1.96$
$\mathrm{m}=1.52535 \ldots$
$\mathrm{w}=1.96-\mathrm{m} \quad \mathrm{w}=.43464 \ldots$
Radian mode.
Area $=(2 x) w-\left(r^{2} / 2\right)(\theta-\sin \theta)$
Area $=.339$

